Eur. Phys. J. B **22**, 455–459 (2001)

THE EUROPEAN PHYSICAL JOURNAL B

EDP Sciences

Magnetic scattering effects on magnetoresistance in a quasi-two-dimensional disordered electron system

Y.H. Yang[1,2,a], Y.G. Wang[1], and M. Liu[1]

[1] Department of Physics, Southeast University, Nanjing 210096, PR China
[2] Laboratory of Solid State Microstructures, Nanjing University, Nanjing 210008, PR China

Received 20 March 2001 and Received in final form 28 June 2001

Abstract. Magnetic-impurity-scattering effects in a quasi-2D disordered electron system have been investigated theoretically with the diagrammatic techniques in perturbation theory. The analytical expressions for magnetoconductivities due to weak-localization effects have been obtained as functions of elastic, inelastic and magnetic scattering times. The relevant dimensional crossover behavior from 3D to 2D with decreasing the interlayer coupling has been discussed, and the condition for the crossover has been obtained.

PACS. 73.20.Fz Weak or Anderson localization – 72.15.Rn Localization effects (Anderson or weak localization) – 73.50.Bk General theory, scattering mechanisms

1 Introduction

Anderson localization of disordered electron systems by elastic scattering from static impurities has been a topic of serious study for the last two decades [1,2]. According to the scaling theory of the pioneering work of Abrahams *et al.* [3], all electronic states in one- and two-dimensional (1D and 2D) disordered systems are localized irrespective of the degree of randomness, while in three-dimensional (3D) systems there exist metal-insulator transitions due to Anderson localization. In recent years, however, quasi-2D electron systems have attracted a great deal of attention because of their unique physical properties. A positive magnetoresistance due to suppression of antilocalization in a $CdTe/Hg_{1-y}Cd_yTe$ superlattice has been studied experimentally by Moyle *et al.* [4]. Szott *et al.* have completed the measurements and made extended studies of negative magnetoresistance effects in a $GaAs/Al_xGa_{1-x}As$ superlattice [5]. Another example of quasi-2D electron system is the layered high-T_c cuprates. The logarithmic increase of resistivity with decreasing temperature in a magnetic field suppressing superconductivity in $La_{2-x}Sr_xCuO_4$ [6] and La-doped $Bi_2Sr_2CuO_7$ [7], is attributed to weak-localization effects [2]. These experimental results provide a motive for theoretical investigation of weak-localization effects in quasi-2D disordered electron systems [8–12]. In a recent work [12], Abrikosov calculated the quantum interference corrections in a quasi-2D metal to conductivity as a function of temperature and magnetic field , and discussed the dimensional crossover from 3D to 2D behavior with decreasing the interlayer hopping energy.

Weak-localization is a quantum effect that results from constructive interference between closed electron paths and their time-reversed counterparts. This constructive interference increases the probability of backscattering and results in an increase in resistivity over the classical Drude value. In this work, we will study theoretically the magnetic-impurity-scattering effects on weak-localization in a quasi-2D disordered electron system, which were not involved in above-mentioned theoretical works. Magnetic impurities introduce an interaction with conduction electrons, and scatter the two complementary electronic waves differently and destroy their coherence after the magnetic scattering time. Therefore magnetic scatterings must have very important influences on the transport properties of a quasi-2D system, as well as on the dimensional crossover behavior from 3D to 2D. By means of the diagrammatic techniques in the perturbation theory, we will calculate the magnetoresistance due to weak-localization effects in a quasi-2D disordered electron system in the presence of magnetic-impurity scatterings, and discuss the relevant dimensional crossover behavior from 3D to 2D with decreasing the interlayer hopping energy.

In Section 2, we will present the model for a layered quasi-2D disordered electron system, and calculate Boltzmann conductivities of this model. In perturbation theory, the so-called Cooperon(particle-particle propagator) is responsible for weak-localization effects, therefore we will, in Section 3, derive the expression for Cooperon

[a] e-mail: ygwang@seu.edu.cn

in the presence of magnetic-impurity scatterings in a magnetic field perpendicular to the layers. The evaluation for weak-localization corrections to conductivities will be presented in Section 4. Finally a brief summary is given in Section 5.

2 The model for a quasi-2D disordered electron system

Let us consider a quasi-2D disordered electron system with the following energy spectrum

$$\epsilon_{\mathbf{k}} = k_{\|}^2/2m - t\cos(k_z a) \tag{1}$$

where $\mathbf{k}_{\|} = (k_x, k_y)$ and k_z are wavevectors along the planar and z-directions respectively, m is the in-plane effective mass, a is the period of the structure along z-axis, and t is the interlayer hopping energy which is assumed to be much smaller than the Fermi energy $\epsilon_{\rm F}$. It is easily shown that the Fermi surface of this model is a slightly corrugated cylinder, the density of states per spin at the Fermi energy is $N = m/2\pi a$, and the electron density is given by $n = k_{\rm F}^2/2\pi a$ with $k_{\rm F} = mv_{\rm F} = \sqrt{2m\epsilon_{\rm F}}$.

We assume that the normal and magnetic impurities introduce the interactions with conduction electrons $u\delta(\mathbf{r})$ and $J\delta(\mathbf{r})\boldsymbol{s}\cdot\boldsymbol{\sigma}$ respectively, with $\boldsymbol{s}$ and $\boldsymbol{\sigma}$ being the spin operators of the magnetic impurity and the conduction electron respectively. The impurity-averaged retarded and advanced Green's functions for the conduction electrons are given by

$$G^{\rm RA}(\mathbf{k}, \omega) = (\omega - \xi_{\mathbf{k}} \pm {\rm i}/2\tau)^{-1} \tag{2}$$

where $\xi_k = \epsilon_{\mathbf{k}} - \epsilon_{\rm F}$ and $\tau^{-1} = \tau_0^{-1} + \tau_{\rm i}^{-1} + \tau_{\rm s}^{-1}$, with τ_0, $\tau_{\rm i}$ and $\tau_{\rm s}$ being the elastic, inelastic and magnetic scattering times respectively. Using the Born approximation, we have $\tau_0^{-1} = 2\pi N n_{\rm i} u^2$ and $\tau_{\rm s}^{-1} = 2\pi N n_{\rm s} J^2 s(s+1)$ with $n_{\rm i}$ and $n_{\rm s}$ being the concentrations of normal and magnetic impurities respectively [2]. The inelastic scattering time $\tau_{\rm i}$ depends on the temperature due to electron-electron or electron-phonon interactions. In the weak-disorder regime, $n_{\rm i}$ and $n_{\rm s}$ are assumed to be so small that $\epsilon_{\rm F}^{-1} \ll \tau_0 \ll \tau_{\rm s}, \tau_{\rm i}$.

The diffusion constant and the mean free path along μ direction are defined by $D_\mu = \langle v_\mu^2 \rangle_{\rm F} \tau$ and $l_\mu = (D_\mu \tau)^{1/2}$ respectively, where $\langle v_\mu^2 \rangle_{\rm F}$ represents the mean-square velocity on the Fermi surface. Making use of the dispersion relation (1), one can easily obtain $D_{\|} = v_{\rm F}^2 \tau/2$ and $D_z = t^2 a^2 \tau/2$. The Boltzmann *dc* conductivities can be easily calculated through the well-known Einstein relation $\sigma_\mu = 2{\rm e}^2 N D_\mu$, yielding $\sigma_{\|} = n{\rm e}^2\tau/m$ and $\sigma_z = {\rm e}^2 mat^2\tau/2\pi$.

It is important to emphasize that both Boltzmann theory and weak-localization theory are correct within the region that quasiclassical approximation is valid. Therefore we must distinguish two different cases: (i) $\tau^{-1} \ll t \ll \epsilon_{\rm F}$, meaning $l_{\|} \gg \lambda_{\rm F}$(the Fermi wavelength) and $l_z \gg a$, in this case the quasiclassical method is valid for all directions; (ii) $t \lesssim \tau^{-1} \ll \epsilon_{\rm F}$, meaning $l_{\|} \gg \lambda_{\rm F}$ and $l_z \lesssim a$, in this case the quasiclassical method is valid only for the planar direction, with the wave functions of electrons being localized along z-direction.

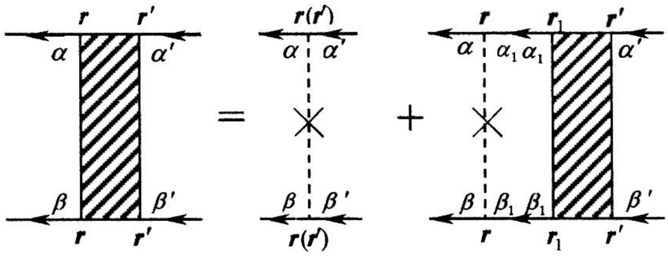

Fig. 1. Diagrams for the Cooperon.

3 The Cooperon in the presence of magnetic scatterings and the magnetic field

Let us consider an external magnetic field perpendicular to the layers, which is described by the vector potential $\mathbf{A} = (-Hy, 0, 0)$. We assume that the field is weak enough so that $\tau \ll \tau_H$ with $\tau_H = c/4eHD_{\|}$, which means that the in-plane mean free path is much smaller than the cyclotron radius. It is favorable to perform the calculation in real space instead of momentum space. The vector potential modifies the phase of the wave functions of electrons which results in a partial destruction of the quantum interference. Then the Green's function in the presence of the magnetic field is given by [13]

$$\tilde{G}^{\rm RA}(\mathbf{r},\ \mathbf{r}^{'};\ \omega) = G^{\rm RA}(\mathbf{r},\ \mathbf{r}^{'};\ \omega)\exp\left[{\rm i}e\int_{\mathbf{r}}^{\mathbf{r}^{'}} \mathbf{A}(\boldsymbol{s})\cdot {\rm d}\boldsymbol{s}\right] \tag{3}$$

where the integral is along a straight line connecting $\mathbf{r}$ and $\mathbf{r}^{'}$. The Cooperon responsible for weak-localization effects is the particle-particle propagator, which can be diagrammatically represented as in Figure 1. The dashed lines with crosses represent the impurity-averaged amplitude, which can be expressed by [2]

$$W_{\alpha\alpha',\ \beta\beta'} = (2\pi N\tau_0)^{-1}\left[\delta_{\alpha\alpha'}\delta_{\beta\beta'} + (\tau_0/3\tau_{\rm s})\boldsymbol{\sigma}_{\alpha\alpha'}\cdot\boldsymbol{\sigma}_{\beta\beta'}\right] \tag{4}$$

where $\sigma^\mu(\mu = x,\ y,\ z)$ are the Pauli matrices, and the first and second terms in equation (4) correspond to normal and magnetic impurity scatterings respectively. The Cooperon is decided by the following equation

$$C(\mathbf{r},\ \mathbf{r}^{'};\ \omega)_{\alpha\alpha',\ \beta\beta'} = W_{\alpha\alpha',\ \beta\beta'}\delta(\mathbf{r}-\mathbf{r}^{'}) + \sum_{\alpha_1\beta_1}\int {\rm d}^3 r_1 W_{\alpha\alpha_1,\ \beta\beta_1} K(\mathbf{r},\ \mathbf{r}_1;\ \omega) C(\mathbf{r}_1,\ \mathbf{r}^{'};\ \omega)_{\alpha_1\alpha',\ \beta_1\beta'} \tag{5}$$

where the kernel $K(\mathbf{r},\ \mathbf{r}_1;\ \omega)$ is defined by

$$K(\mathbf{r},\ \mathbf{r}_1;\ \omega) = \tilde{G}^{\mathrm{R}}(\mathbf{r},\ \mathbf{r}_1;\ \omega)\ \tilde{G}^{\mathrm{A}}(\mathbf{r},\ \mathbf{r}_1;\ 0). \quad (6)$$

In order to calculate $C(\mathbf{r},\ \mathbf{r}';\ \omega)_{\alpha\alpha',\ \beta\beta'}$ in equation (5), we shall try to solve the following integral equation

$$\int \mathrm{d}^3 r' K(\mathbf{r},\ \mathbf{r}';\ \omega)\psi(\mathbf{r}') = K(\omega)\psi(\mathbf{r}). \quad (7)$$

If ω is so small that $\omega\tau \ll 1$, one can solve equation (7) with the similar method as in a purely 2D system [13], obtaining the eigenfunctions and eigenvalues of the kernel $K(\mathbf{r},\ \mathbf{r}';\ \omega)$ as follows

$$\psi_{nq_xq_z}(\mathbf{r}) = \exp(\mathrm{i}q_x x)\psi_n(y - cq_x/2eH)\phi_{q_z}(z) \quad (8)$$

$$K(n,\ q_z;\ \omega) = 2\pi N\tau\Big[1 + \mathrm{i}\omega\tau - (n+1/2)\tau/\tau_H - (2l_z/a)^2\sin^2(q_z a/2)\Big] \quad (9)$$

where ψ_n is the eigenfunction of an oscillator, and $\phi_{q_z}(z)$ is the Bloch wave function along z-direction. Now we can expand $K(\mathbf{r},\ \mathbf{r}';\ \omega)$ and $C(\mathbf{r},\ \mathbf{r}';\ \omega)_{\alpha\alpha',\ \beta\beta'}$ in terms of the eigenfunctions (8), obtaining

$$K(\mathbf{r},\ \mathbf{r}';\ \omega) = \sum_{nq_xq_z} K(n,\ q_z;\ \omega) \times \psi_{nq_xq_z}(\mathbf{r})\psi^*_{nq_xq_z}(\mathbf{r}') \quad (10)$$

$$C(\mathbf{r},\ \mathbf{r}';\ \omega)_{\alpha\alpha',\ \beta\beta'} = \sum_{nq_xq_z} C(n,\ q_z;\ \omega)_{\alpha\alpha',\ \beta\beta'} \times \psi_{nq_xq_z}(\mathbf{r})\psi^*_{nq_xq_z}(\mathbf{r}'). \quad (11)$$

Substituting equations (10, 11) into equation (5), we obtain

$$C(n,\ q_z;\ \omega)_{\alpha\alpha',\ \beta\beta'} = W_{\alpha\alpha',\ \beta\beta'} + K(n,\ q_z;\ \omega) \times \sum_{\alpha_1\beta_1} W_{\alpha\alpha_1,\ \beta\beta'} C(n,\ q_z;\ \omega)_{\alpha_1\alpha',\ \beta_1\beta'}. \quad (12)$$

We expect that the expression for Cooperon has the same structure as the scattering amplitude, assuming

$$C(n,\ q_z;\ \omega)_{\alpha\alpha',\ \beta\beta'} = (2\pi N\tau)^{-1} \times \Big(A\delta_{\alpha\alpha'}\delta_{\beta\beta'} + B\boldsymbol{\sigma}_{\alpha\alpha'}\cdot\boldsymbol{\sigma}_{\beta\beta'}\Big). \quad (13)$$

Substituting equations (4, 9, 13) into equation (12), one can calculate the values of A and B, yielding

$$\begin{aligned}\sum_{\alpha\beta} C(n,\ q_z;\ \omega)_{\alpha\beta,\ \beta\alpha} &= (2\pi N\tau)^{-1}(2A+6B)\\ &= (2\pi N\tau)^{-1}\times\\ \Bigg(3\Big[(n+1/2)\tau/\tau_H &+ (2l_z/a)^2\sin^2(q_z a/2) - \mathrm{i}\omega\tau + \lambda_1\Big]^{-1}\\ - \Big[(n+1/2)\tau/\tau_H &+ (2l_z/a)^2\sin^2(q_z a/2) - \mathrm{i}\omega\tau + \lambda_2\Big]^{-1}\Bigg)\end{aligned} \quad (14)$$

where $\lambda_1 = \tau/\tau_{\mathrm{i}} + 2\tau/3\tau_{\mathrm{s}}$, and $\lambda_2 = \tau/\tau_{\mathrm{i}} + 2\tau/\tau_{\mathrm{s}}$.

Equation (14) is the expression for Cooperon which has a different form from that of a 3D system with anisotropic effective masses due to the special structure of the energy spectrum in the quasi-2D system, and will be used in the following calculation.

4 Magnetoresistance due to weak-localization effects

The calculation for conductivities in the quasiclassical approximation can be easily performed by means of the Kubo formalism. In the presence of a magnetic field, the quantum interference correction to the conductivity along μ-direction is given by [2,13].

$$\sigma_\mu{}^{[\mathrm{WL}]} = (e^2/2\pi)\sum_{\mathbf{k}}\sum_{nq_xq_z}\sum_{\alpha\beta} v_\mu(\mathbf{k})v_\mu(-\mathbf{k}) \times |G^{\mathrm{R}}(\mathbf{k},0)|^4 C(n,\ q_z;\ \omega)_{\alpha\beta,\ \beta\alpha} \quad (15)$$

where $\omega \ll \tau^{-1}$ is the frequency of the applied field, and $v_\mu(\mathbf{k}) = \partial\epsilon_{\mathbf{k}}/\partial k_\mu$ is the velocity of electrons along μ-direction. We can easily perform the integrations over $\mathbf{k}$ and q_x, getting

$$\frac{\sigma_\mu{}^{[\mathrm{WL}]}}{\sigma_\mu} = -\frac{eH\tau^2}{\pi c}\sum_{n=0}^{\tau_H/\tau}\sum_{q_z}\sum_{\alpha\beta} C(n,\ q_z;\ \omega)_{\alpha\beta,\ \beta\alpha}. \quad (16)$$

Substituting equation (14) into equation (16), we get the general expression for the relative corrections to conductivities as

$$\begin{aligned}\frac{\sigma_\mu{}^{[\mathrm{WL}]}}{\sigma_\mu} = -\frac{\omega_{\mathrm{c}}\tau a}{\pi}\sum_{n=0}^{\tau_H/\tau}\sum_{q_z}\Bigg(3\Big[(n+\frac{1}{2})\frac{\tau}{\tau_H} &+ \frac{4l_z^2}{a^2}\sin^2(\frac{1}{2}q_z a) - \mathrm{i}\omega\tau + \lambda_1\Big]^{-1}\\ - \Big[(n+\frac{1}{2})\frac{\tau}{\tau_H} + \frac{4l_z^2}{a^2}\sin^2(\frac{1}{2}q_z a) &- \mathrm{i}\omega\tau + \lambda_2\Big]^{-1}\Bigg)\end{aligned} \quad (17)$$

where $\omega_{\mathrm{c}} = eH/mc$ is the cyclotron frequency. In order to discuss the dimensional crossover behavior from 3D to 2D, we will set $\omega = 0$ and study several limiting cases.

If the interlayer hopping energy t is large enough so that $\tau^{-1} \ll t \ll \epsilon_{\mathrm{F}}$, meaning $l_\parallel \gg \lambda_{\mathrm{F}}$ and $l_z \gg a$, then the quasiclassical approximation is valid for all directions, and the main contribution of equation (17) arises from $|q_z| \ll l_z^{-1}$. Replacing the summation over q_z by the integral $\int_{-l_z^{-1}}^{l_z^{-1}} \frac{\mathrm{d}q_z}{2\pi}$, we get

$$\frac{\sigma_\mu{}^{[\mathrm{WL}]}(H)}{\sigma_\mu} = -\frac{\omega}{\sqrt{2}\pi t}\sqrt{\frac{\tau_H}{\tau}}\sum_{n=0}^{\tau_H/\tau}\Bigg[3\Big(n+\frac{1}{2}+\lambda_1\frac{\tau_H}{\tau}\Big)^{-1/2} - \Big(n+\frac{1}{2}+\lambda_2\frac{\tau_H}{\tau}\Big)^{-1/2}\Bigg]\cdot \quad (18)$$

When $H \to 0$, meaning $\tau_H \to \infty$, one can replace the summation over n in equation (18) by an integration, obtaining

$$\frac{\sigma_\mu{}^{[\mathrm{WL}]}(0)}{\sigma_\mu} = -\frac{1}{\sqrt{2}\pi t\epsilon_{\mathrm{F}}\tau^2}\Big[3(\sqrt{1+\lambda_1}-\sqrt{\lambda_1}) - (\sqrt{1+\lambda_2}-\sqrt{\lambda_2})\Big]. \quad (19)$$

Combining equations (18, 19), we obtain the magnetoconductivities due to weak-localization effects as

$$\frac{\Delta\sigma_\mu(H)}{\sigma_\mu} = \frac{\sigma_\mu{}^{[\mathrm{WL}]}(H)-\sigma_\mu{}^{[\mathrm{WL}]}(0)}{\sigma_\mu} = \frac{\omega_{\mathrm{c}}}{\sqrt{2}\pi t}\sqrt{\frac{\tau_H}{\tau}}\left[3f(\lambda_1\frac{\tau_H}{\tau}) - f(\lambda_2\frac{\tau_H}{\tau})\right] \quad (20)$$

where the function $f(x)$ is defined by

$$f(x) = \sum_{n=0}^{\infty}\left[2(n+1+x)^{1/2} - 2(n+x)^{1/2} - (n+\frac{1}{2}+x)^{-1/2}\right].$$

The dependence of magnetic field in equation (20) is the characteristic behavior of a 3D system [14].

If the interlayer hopping energy t is small enough so that $t \lesssim \tau^{-1} \ll \epsilon_{\mathrm{F}}$, meaning $l_\parallel \gg \lambda_{\mathrm{F}}$ and $l_z \lesssim a$, then the quasiclassical approximation is valid only for the planar direction. Replacing the summation over q_z in equation (17) by the integral $\int_{-\pi/a}^{\pi/a}\frac{\mathrm{d}q_z}{2\pi}$, we can get

$$\frac{\sigma_\parallel{}^{[\mathrm{WL}]}(H)}{\sigma_\parallel} = -\frac{1}{4\pi\epsilon_{\mathrm{F}}\tau} \times \sum_{n=0}^{\tau_H/\tau}\left(3\left[\left(n+\frac{1}{2}+\lambda_1\frac{\tau_H}{\tau}+t^2\tau\tau_H\right)^2-(t^2\tau\tau_H)^2\right]^{-1/2} - \left[\left(n+\frac{1}{2}+\lambda_2\frac{\tau_H}{\tau}+t^2\tau\tau_H\right)^2-(t^2\tau\tau_H)^2\right]^{-1/2}\right). \quad (21)$$

When $H \to 0$, meaning $\tau_H \to \infty$, we can replace the summation over n in equation (21) by an integration, obtaining

$$\frac{\sigma_\parallel{}^{[\mathrm{WL}]}(0)}{\sigma_\parallel} = -\frac{1}{4\pi\epsilon_{\mathrm{F}}\tau} \times\left[3\ln\frac{1+\lambda_1+t^2\tau^2+\sqrt{(1+\lambda_1+t^2\tau^2)^2-(t^2\tau^2)^2}}{\lambda_1+t^2\tau^2+\sqrt{(\lambda_1+t^2\tau^2)^2-(t^2\tau^2)^2}} - \ln\frac{1+\lambda_2+t^2\tau^2+\sqrt{(1+\lambda_2+t^2\tau^2)^2-(t^2\tau^2)^2}}{\lambda_2+t^2\tau^2+\sqrt{(\lambda_2+t^2\tau^2)^2-(t^2\tau^2)^2}}\right]. \quad (22)$$

Combining equations (21, 22), one can get the expression for the magnetoconductivity along the planar direction as

$$\frac{\Delta\sigma_\parallel(H)}{\sigma_\parallel} = \frac{1}{4\pi\epsilon_{\mathrm{F}}\tau}\Big[3F(\lambda_1\frac{\tau_H}{\tau},\ t^2\tau\tau_H) - F(\lambda_2\frac{\tau_H}{\tau},\ t^2\tau\tau_H)\Big] \quad (23)$$

where the function $F(x,\ y)$ is defined by

$$F(x,\ y) = \sum_{n=0}^{\infty}\left(\ln\frac{n+1+x+y+\sqrt{(n+1+x+y)^2-y^2}}{n+x+y+\sqrt{(n+x+y)^2-y^2}} - \left[(n+\frac{1}{2}+x+y)^2-y^2\right]^{-1/2}\right) \approx \psi\Big(\frac{1}{2}+x\Big) - \ln x, \qquad \text{if } x \gg y$$

with $\psi(x)$ being the well-known digamma function.

In the case of $t \ll \max\left\{(\tau\tau_{\mathrm{i}})^{-1/2},\ (\tau\tau_{\mathrm{s}})^{-1/2}\right\}$, meaning $\lambda_l\tau_H/\tau \gg t^2\tau\tau_H,\ (l=1,\ 2)$, equation (23) changes as

$$\frac{\Delta\sigma_\parallel(H)}{\sigma_\parallel} = \frac{1}{4\pi\epsilon_{\mathrm{F}}\tau}\left[3\psi(\frac{1}{2}+\lambda_1\frac{\tau_H}{\tau}) - 3\ln(\lambda_1\frac{\tau_H}{\tau}) - \psi(\frac{1}{2}+\lambda_2\frac{\tau_H}{\tau}) + \ln(\lambda_2\frac{\tau_H}{\tau})\right] \quad (24)$$

which is the exact result found in a purely 2D system [2]. From equations (20, 23, 24), one can see that there exists a dimensional crossover behavior from 3D to 2D with decreasing the interlayer hopping energy.

It is necessary to point out that equation (15) corresponds to the conductivity diagram with one Cooperon [2,13], meaning that only the lowest-order quantum correction is concerned. This approximation is valid for the weak-disorder regime ($l_\parallel \gg \lambda_{\mathrm{F}}$) considered throughout this paper, which is far away from the metal-insulator transition [1]. As the degree of randomness increases, especially near the metal-insulator transition, the higher-order quantum corrections are not negligible. In this case the two-loop diagrams with effectively two diffusion propagators have to be taken into account [15–17], which exceeds the range of this paper.

5 Conclusions

In this work, we have investigated the magnetic-impurity-scattering effects in a quasi-2D disordered electron system. By means of the diagrammatic techniques in the perturbation theory, we have calculated the magnetoresistances due to weak-localization effects. The analytical results for the magnetoconductivities have been obtained as functions of three characteristic times: elastic, inelastic and magnetic scattering times. We show that all these scattering times have very important influences on the relevant dimensional crossover from 3D to 2D. In the 3D limit of $t \gg \tau^{-1}$, the relative magnetoconductivities due to weak-localization effects are independent of directions, and have the similar dependence of field with that of an isotropic 3D system. If the interlayer coupling t is small enough so that $t \lesssim \tau^{-1}$, the quasiclassical approximation for transport properties is not valid for z-direction and the planar magnetoconductivity has very complex dependence of the magnetic field (see Eq. (23)). In the 2D case of $t \ll \max\left\{(\tau\tau_{\rm i})^{-1/2}, (\tau\tau_{\rm s})^{-1/2}\right\}$, the planar magnetoconductivity is exactly the same as in an isotropic 2D system. Therefore, the relevant dimensional crossover from 3D to 2D occurs in the region of $\max\left\{(\tau\tau_{\rm i})^{-1/2}, (\tau\tau_{\rm s})^{-1/2}\right\} \lesssim t \lesssim \tau^{-1}$.

This work was supported by the Natural Science Foundation of China Grant 19874011.

References

1. P.A. Lee, T.V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985).
2. G. Bergmann, Phys. Rep. **107**, 1 (1984).
3. E. Abrahams, P.W. Anderson, D.C. Licciardello, T.V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979).
4. J.K. Moyle, J.T. Cheung, N.P. Ong, Phys. Rev. B **35**, 5639 (1987).
5. W. Szott, C. Jedrzejek, W.P. Kirk, Phys. Rev. Lett. **63**, 980 (1989).
6. Y. Ando, G.S. Boebinger, A. Passner, T. Kimura, K. Kishio, Phys. Rev. Lett. **75**, 4662 (1995).
7. Y. Ando, G.S. Boebinger, A. Passner, N.L. Wang, C. Geibel, F. Steglich, Phys. Rev. Lett. **77**, 2065 (1996).
8. W. Szott, C. Jedrzejek, W.P. Kirk, Phys. Rev. B **40**, 1790 (1989).
9. X.J. Lu, N.J.M. Horing, Phys. Rev. B **44**, 5651 (1991).
10. Y.H. Yang, D.Y. Xing, C.D. Gong, J. Phys. Cond. Matt. **5**, 6231 (1993).
11. A.A. Abrikosov, Phys. Rev. B **50**, 1415 (1994).
12. A.A. Abrikosov, Phys. Rev. B **61**, 7770 (2000).
13. B.L. Altshuler, D. Khmel'nitzkii, A.I. Larkin, P.A. Lee, Phys. Rev. B **22**, 5142 (1980).
14. A. Kawabata, Solid State Commun. **34**, 431 (1980).
15. S. Hikami, Phys. Rev. B **24**, 2671 (1981).
16. A. Kawabata, J. Phys. Soc. Jpn **53**, 318 (1984).
17. C.S. Ting, Phys. Rev. B **31**, 3765 (1985).